Total No. of Questions—**8**]

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Seat No.	

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S.E. (Comp./IT) (II Semester) EXAMINATION, 2017

ENGINEERING MATHEMATICS—III

(2012 PATTERN)

Time : Two Hours **Maximum Marks : 50**

***N.B.* :—** (*i*) Attempt Q. No. **1** *or* Q. No. **2**, Q. No. **3** *or* Q. No. **4**, Q. No. **5** *or* Q. No. **6**, Q. No. **7** *or* Q. No. **8**.

(*ii*) Neat diagrams must be drawn wherever necessary.

(*iii*) Figures to the right indicate full marks.

(*iv*) Use of logarithmic tables, slide rule, Mollier charts, electronic pocket calculator and steam tables is allowed.

(*v*) Assume suitable data, if necessary.

1. (*a*) Solve any *two* : [8]

(*i*) $(D^2 + 1)y = x \cos x$

(*ii*) $(D^2 - 4D + 4)y = 8x^2 e^{2x} \sin x$

(*iii*) $(D^2 + 9)y = \dfrac{1}{1 + \sin 3x}$.

(*b*) Find the Fourier sine transform of $f(x) = \dfrac{e^{-ax}}{x}$. [4]

P.T.O.

Or

2. (*a*) An e.m.f. E sin pt is applied at $t = 0$ to a circuit containing a condenser C and inductance L in series the current I satisfies the equation : [4]

$$L\frac{dI}{dt} + \frac{1}{C}\int I\,dt = E \sin pt, \text{ where}$$

$$i = -\frac{dq}{dt}, \text{ If } p^2 = \frac{1}{LC}$$

and initially the current and the charge are zero, find current at any time t.

(*b*) Find the inverse z-transform (any *one*) : [4]

(*i*) $F(z) = \dfrac{z}{(z-1)(z-2)}, |z| > 2$

(*ii*) $F(z) = \dfrac{1}{(z-2)(z-3)}, 2 < |z| < 3$

(*c*) Solve the following differnece equation to find $f(k)$: [4]

$$6f(k+2) - 5f(k+1) + f(k) = 0$$

$$f(0) = 0, f(1) = 3, k \geq 0.$$

3. (*a*) The first four moments of a distribution about 25 are –1.1, 89, –110 and 23300. Calculate the first four moments about the mean. [4]

(*b*) In a Poisson distribution if : [4]

$$P(r = 1) = 2\ P(r = 2)$$

then show that :

$$P(r = 3) = 0.0613.$$

(*c*) Find the directional derivative of $\phi = 4xz^3 - 3x^2y^2z$ at (2, –1, 2) towards the point $\bar{i} + \bar{j} - \bar{k}$. [4]

Or

4. (*a*) Attempt any *one* : [4]

(*i*) For scalar functions ϕ and ψ, show that :

$$\nabla . (\phi \nabla \psi - \psi \nabla \phi) = \phi \nabla^2 \psi - \psi \nabla^2 \phi.$$

(*ii*) Show that :

$$\nabla^2 \left(\frac{\bar{a} \, . \, \bar{b}}{r} \right) = 0.$$

(*b*) Show that the vector field :

$$\bar{F} = (ye^{xy} \cos z) \, \bar{i} + (xe^{xy} \cos z) \, \bar{j} + (-e^{-xy} \sin z) \bar{k}$$

is irrotational. Also find the corresponding scalar ϕ, such that $\bar{F} = \nabla \phi$. [4]

(*c*) If the two lines of regression are $9x + y - \lambda = 0$ and $4x + y - \mu = 0$ and the means of x and y are 2 and –3 respectively, then find λ, μ and the coefficient of correlation between x and y. [4]

5. (*a*) Apply Green's Lemma to evaluate the : [5]

$$\oint_C (3x^2 - 8y^2) \, dx + (4y - 6xy) \, dy,$$

where C is the boundary of the region defined by $y = \sqrt{x}$, $y = x^2$ in the plane $z = 0$.

(*b*) If : [4]

$$\overline{\mathrm{F}} = (x^2 + y - 4)i + 3xy\hat{j} + (2xz + z^2)\hat{k},$$

evaluate :

$$\iint_S (\nabla \times \overline{\mathrm{F}}) \cdot \hat{n}\ d\mathrm{S},$$

where S is the surface of the sphere :

$$x^2 + y^2 + z^2 = 16$$

above the xy plane.

(*c*) Evaluate : [4]

$$\iint_S \overline{\mathrm{F}} \cdot \hat{n}\ d\mathrm{S},$$

where

$$\overline{\mathrm{F}} = (2x + 3y^2z^2)i - (x^2z^2 + y)\hat{j} + (y^3 + 2z)\hat{k}$$

and S is the surface of the sphere with centre (3, –1, 2) and radius 3.

Or

6. (*a*) Evaluate $\int_C \overline{\mathrm{F}} \cdot d\overline{r}$ from the point (0, 0, 0) to (1, 1, 1) along the curve $x = t$, $y = t^2$, $z = t^3$, given : [4]

$$\overline{\mathrm{F}} = xy\hat{i} - z^2\hat{j} + xyz\hat{k}.$$

(*b*) Using divergence theorem, evaluate $\int_S \overline{\mathrm{F}} \cdot \hat{n}\ d\mathrm{S}$, over S, the surface of unit cube bounded by the co-ordinates planes and the planes $x = 1$, $y = 1$ and $z = 1$ where $\overline{\mathrm{F}} = 2xi + 3y\hat{j} + 4z\hat{k}$. [4]

(*c*) Apply Stokes' theorem to evaluate $\int_C \bar{F} . d\bar{r}$, where $\bar{F} = y\hat{i} + zj + x\hat{k}$, where C is the curve given by : [5]

$$x^2 + y^2 + z^2 - 2ax - 2ay = 0$$

and

$$x + y = 2a.$$

7. (*a*) If $v = 4xy(x^2 - y^2)$, find u such that $f(z) = u + iv$ is analytic and determine $f(z)$ in terms of z. [4]

(*b*) Evaluate $\oint_C \tan z \, dz$ where C is the circle $|z| = 2$. [5]

(*c*) Show that the transformation $W = \frac{1}{z}$ maps the circle $x^2 + y^2 - 6x = 0$ onto a straight line in W-plane. [4]

Or

8. (*a*) If $f(z) = u + iv$ is analytic and $u + v = \sin x . \cosh y + \cos x . \sinh y$, then find $f(z)$ is terms of z. [4]

(*b*) Evaluate $\oint_C \frac{e^z}{(z-1)^2 (z-2)} dz$ where 'C' is the contour $|z - 2| = \frac{3}{2}$ by using Cauchy's residue theorem. [5]

(*c*) Find the bilinear transformation which maps the points $0, \frac{1}{2}, 1 + i$ from z-plane into the points $-4, \infty, 2 - 2i$. [4]